MoSAIC Evaluation

LAAS-CNRS
GSPN Model for \((k,n)\) Erasure Code

- **OU** (owner up)
- **FC** (fragments to create)
- **MF** (mobile fragments)
- **SF** (safe fragments)
- **DL** (data lost)
- **DS** (data safe)

Transitions:
- \(\lambda\): owner fails
- \(\beta\): owner meets infrastructure
- \(m(MF)\cdot\beta'\): contributor meets infrastructure
- \(m(MF)\cdot\lambda'\): contributor fails

Properties:
- \(m(MF) + m(SF) < k\)
- \(m(SF) \geq k\)
Probability of data loss

• PL(k,n) : probability of data being lost
• Without MoSAIC

\[ PL(0,0) = \frac{\lambda}{\lambda + \beta} \]

• With MoSAIC

\[ PL(1,1) = \frac{\lambda}{\lambda + \beta + \alpha} + \alpha \left( \frac{\lambda}{\lambda + \beta} \right)^2 \]

\[ PL(1,2) = \frac{\lambda}{\lambda + \beta + \alpha} + \frac{\alpha^2}{(\lambda + \beta + \alpha)(\lambda + 2\beta + \alpha)} \left[ \frac{\lambda}{\alpha} \left( \frac{\lambda}{\lambda + \beta + \alpha} + \frac{\lambda}{\lambda + \beta} \right) + \frac{\lambda \alpha}{\lambda + \beta + \alpha} \left( \frac{\lambda}{\lambda + \beta} \right)^2 + \alpha \left( \frac{\lambda}{\lambda + \beta} \right)^3 \right] \]
without MoSAIC

need $\alpha/\beta >> 1$ for significant improvement

for large $\beta/\lambda$, improvement independent of degree of replication
Loss probability improvement

Let \( IPL(k,n) = PL(0,0)/PL(k,n) \) : reduction in probability of data being lost provided by MoSAIC

\[
IPL(1,1) = \left[ \frac{\lambda}{\lambda + \beta + \alpha} + \frac{\alpha}{\lambda + \beta + \alpha} \left( \frac{\lambda}{\lambda + \beta} \right)^2 \right] \times \left( \frac{\lambda + \beta}{\lambda} \right)^{-1}
\]

\[
= \left[ \frac{\lambda + \beta}{\lambda + \beta + \alpha} + \frac{\alpha}{\lambda + \beta + \alpha} \left( \frac{\lambda}{\lambda + \beta} \right) \right]^{-1}
\]

\[
= \left[ \frac{\lambda/\beta + 1}{\lambda/\beta + 1 + \alpha/\beta} + \frac{\alpha/\beta}{\lambda/\beta + 1 + \alpha/\beta} \left( \frac{\lambda/\beta}{1 + \lambda/\beta} \right) \right]^{-1}
\]

\[
\lim_{\lambda/\beta \to 0} (IPL(1,1)) = \left( \frac{1}{1 + \alpha/\beta} \right)^{-1} = 1 + \alpha/\beta
\]

Conjecture that (can demonstrate via Markov graph approximation):

\[
\forall n, \lim_{\lambda/\beta \to 0} (IPL(1,n)) = 1 + \frac{\alpha}{\beta}
\]
maximum improvement = 1 + $\alpha/\beta$

higher redundancy allows maximum improvement to be obtained at lower values of $\beta/\lambda$. 
in this region, rate of improvement depends only on degree of redundancy
IPL limit for large $\alpha$

- Consider case of $\alpha \to \infty$
- Change of variable: $\alpha/\beta = (\alpha/\lambda)(\lambda/\beta)$

$$ IPL(1,1) = \left[ \frac{\lambda/\beta + 1}{\lambda/\beta + 1 + (\alpha/\lambda)(\lambda/\beta)} + \frac{(\alpha/\lambda)(\lambda/\beta)}{\lambda/\beta + 1 + (\alpha/\lambda)(\lambda/\beta)(1 + \lambda/\beta)} \right]^{-1} $$

$$ = \left[ \frac{(\lambda/\alpha)(\lambda/\beta + 1)}{(\lambda/\alpha)(\lambda/\beta) + \lambda/\alpha + \lambda/\beta} + \frac{(\lambda/\beta)}{(\lambda/\alpha)(\lambda/\beta) + \lambda/\alpha + \lambda/\beta(1 + \lambda/\beta)} \right]^{-1} $$

$$ \lim_{\lambda/\alpha \to 0} IPL(1,1) = \left( \frac{\lambda/\beta}{1 + \lambda/\beta} \right)^{-1} = \left( \frac{1 + \lambda/\beta}{\lambda/\beta} \right) = \left( 1 + \frac{\beta}{\lambda} \right) $$

- Conjecture that (can demonstrate via Markov graph approximation):

$$ \lim_{\lambda/\alpha \to 0} IPL(1,n) = \left( 1 + \frac{\beta}{\lambda} \right)^n $$
MoSAIC - replication (k=1)
Preliminary conclusions

- Have shown limiting expressions for pure replication:

\[
\forall n, \lim_{\lambda/\beta \to 0} \left( IPL(1, n) \right) = 1 + \frac{\alpha}{\beta}
\]

\[
\lim_{\lambda/\alpha \to 0} IPL(1, n) = \left( 1 + \frac{\beta}{\lambda} \right)^n
\]

- Future work:
  - study k>1
  - consider \( \lambda_{\text{contributor}} > \lambda_{\text{owner}} \) (contributors may be malicious)
  - take into account energy and other resource limitations