Software Reliability Analysis and Evaluation

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OUTLINE

☞ Motivations
☞ Methods for software reliability analysis and evaluation (Folder 3)
  • Data collection and analysis
    ☞ Data collection and validation
    ☞ Descriptive statistics
    ☞ Trend analysis
  • Software reliability evaluation (Folder 4)
☞ Case studies (Folder 5)
☞ Dependability benchmarking of operating systems (Folder 6)
Why Software Reliability?

☞ Increasing role of software in real life systems
☞ System dependability is more and more synonymous of software reliability
☞ Difficulties in mastering the software development process and in reducing design faults for complex systems
☞ Increasing cost of system non-dependability
☞ Real needs for improving software reliability to improve system dependability and reduce maintenance cost
☞ Dependability requirements are part of system requirements (as important as functional requirements)
☞ Quantification is essential

Objectives of software reliability analysis and evaluation

☞ Manage and improve the reliability of software products
☞ Check the efficiency of development activities
☞ Estimate the “effort” needed to reach a high dependability level
☞ Evaluate the software reliability at the end of validation activities and in operation
☞ Estimate the maintenance effort to “correct” faults activated during development and residual faults in operation

☞ Needs for experimental & analytical methods and techniques to reach these objectives
Software vs hardware reliability

**Hardware**
- Physical faults
- Operational life
- Stable reliability (constant failure rate)
- White-box approach
- Markov models
- Database for components failures

**Software**
- Only design faults
- Development and operation
- Reliability growth (↓ failure rate)
- Usually black-box approach
- Specific models
- Based on data collection

**Failure Rate**

![Failure Rate Diagram](image)

**Measures**

**Static Measures**
- Number of faults
- Fault density
- Complexity measures

**Dynamic Measures**
- Usage profile (environment)
- Characterizing occurrence of failures

**Failure intensity**
- Failure rate
- MTTF
- Reliability

Software Reliability Evaluation - Karama KANOUN - LAAS-CNRS
-supplier point of view

• During development:
  ☞ development follow up
  (failure intensity, fault density)
  ☞ evaluation of software reliability before operation
  (MTTF, pre-operational failure rate)

• During operation
  ☞ product reliability follow up
  (residual failure rate, MTTF)
  ☞ maintenance planning
  (cumulative number of failures)

-users / customers, operational life

  ☞ be confident in the reliability level of the product
  (residual failure rate, MTTF)

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percentage of faults and corresponding MTTF (published by IBM)

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Overview of a global reliability analysis method

1. Data collection
2. Data Validation
3. Descriptive Analyses
4. Trend Analyses
5. Model Application

Objectives of the study
Validation of data

Consequences of failures
Types of faults
Components

Development & Operation

Data set partition

Descriptive Statistics
Reliability Evolution
Reliability Measures

Feedback to the development process

Setting up a data collection process

Some rules

- Define clearly the objectives and the data to be collected
- Motivate and imply people that will be involved
- Simplify the collection process and reduce the number of data items to be collected
  - Support tools
  - Practical organization of people involved
- Record and analyze data in real-time
- Feedback

Origin of collected data

- Internal: recorded during development and validation
- External: by the customers
Data to be collected

- **Background information**
  - Product itself: software size, language, functions, current version, workload
  - Usage environment: verification and validation methods, tools, etc.

- **Data relative to failures and corrections**
  - Date of occurrence, nature of failures, consequences
  - Type of faults, fault location

- **Usually, recorded through**
  - Failure Reports (FR)
  - Correction Reports (CR)

- **Well defined headings, well structured, easy to fill in**

- **Short tick-off questions**

- **Manually or automatically**

**Failure Report (FR)**

**Required Information**
- Serial number (for identification)
- Report editor
- Product reference, version affected (or prototype)
- Date and time of failure occurrence

**Desirable Information**
- Failure occurrence condition
- Failure criticality or consequences
- Affected function or task
- Action proposed (if any)
Correction Report (CR)

Required information
- Serial number (for identification)
- Report editor
- Date of correction
- Correction nature
- Product reference

Desirable Information
- Identification of the modified components

Integration with already existing data collection programs
Importance of training

Data Validation

Objectives
- check the validity and usability of the information recorded
- Keep only genuine software faults in the database

Elimination of:
- Duplicated data (FR reporting of the same failure)
- FR proposing a correction related to an already existing FR (COR)
- False FR (signaling a false or non identified problem)
- FR proposing an improvement (IMPROVE)
- incomplete FRs or FRs containing inconsistent data (Unusable)
- FR related to a hardware failure
- …
Example 1: a telecommunications equipment
(analyzed at LAAS)

☞ 2 146 Failure Reports
☞ Validation ⇒ 1 172 kept in the database

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Example 2: a telephone switching system
(analyzed at LAAS)

☞ 3063 FRs
☞ Validation ⇒ 1853 Software FRs:

- Software: 1853
- Hardware: 195
- Documentation: 165
- Unusable, duplicated, …: 716
- Others: 134

Unusable: 24%
Documentation: 5%
Hardware: 6%
Software: 61%
Others: 4%
Life cycle of an FR

Identification of an abnormal behavior

Database

FR resolved
Creation of a CR

DESCRIPTIVE STATISTICS

☞ Aim: make syntheses of the observed phenomena

☞ Simple analyses
  • Fault typology
  • Fault density of components
  • Failure / fault distribution among software components (new, modified, reused)

☞ Investigation of relationships
  • Fault density / size
  • Fault density / complexity
  • Fault density / life cycle phase
  • Nature of faults / life cycle phases
  • Nature of faults / components
  • Number of components affected by changes made to resolve an FR

☞ Analyses related to the development / debugging process
Analyses related to the development process

Factors affecting time to locate and solve problems

- The more FRs circulating, the more time it takes to handle each one
- Tendency to resolve the easier FRs first, the remaining ones take more time
- Loss of maintainability with continued changes to resolve faults
- Introduction of new faults while resolving the old

Average time to resolve an FR

Modification request time =

Time when the FR is resolved - time when it is created

Measures

- Responsiveness of the field support system
- Complexity of maintenance

Case of the switching system of Example 2

# FRs recorded: 3063

# FRs with record date: 3049

# FRs with record date and resolve date: 2446

Time to resolve an FR
Data pre-processing for reliability analysis

Two kinds of data sets can be extracted from FRs and CRs
- Time to failures (or between failures)

![Diagram showing time between failures]

- Grouped data
  - Number of failures per unit of time, \( n(k) \)
  - Cumulative number of failures \( N(k) \)

Time ?

- Time between failures
  - Execution time
  - Wall clock or Calendar time
  - Number of executions

- Number of failures per unit of time
  - The length of the unit time depends on:
    - Accuracy expected for the dependability measures
    - Number of observed failures
    - Objectives of the study
### Example A

**Times between failures**

Real-time control system (Musa 1)

- 136 failures observed during system test (96 days)

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### Example B

**Number of failures per unit of time or Cumulative**

Switching system

- 52 failures in operation (15 months)

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<td>13</td>
</tr>
</tbody>
</table>

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Trend analysis

☞ Objectives:
- Analyze software reliability evolution
- Identify periods of reliability growth and decrease

Variable: time between failures
- $T_1, T_2, \ldots, T_n$: time between failure $i$ and $i-1$
- Reliability growth: $T_i \leq T_k$ $\forall$ $i < k$
- $\text{Prob} \{ T_i < x \} \geq \text{Prob} \{ T_k \leq x \}$ $\forall$ $i < k$ $\forall$ $x$

Variable: number of failures
- $N(t_1), N(t_2), \ldots, N(t_n)$: cumulative number of failures between 0 and $t_i$
- $H(t_i) = \text{E}[N(t_i)] = \text{expectation of } N(t_i)$
- If $N(t_i)$ is a Non Homogeneous Poisson Process (NHPP):
  - Reliability growth if $H(t_1) + H(t_2) \geq H(t_1 + t_2)$ $\forall$ $t_1, t_2 \geq 0$ and $0 \leq t_1 + t_2 \leq T$ and inequality is strict for at least a pair $t_1, t_2$
  - $N(t)$ is a subadditive function
  - Reliability decrease if $H(t_1) + H(t_2) \leq H(t_1 + t_2)$ $\forall$ $t_1, t_2 \geq 0$ and $0 \leq t_1 + t_2 \leq T$ and inequality is strict for at least a pair $t_1, t_2$
  - $N(t)$ is a superadditive function
Interpretation of Subadditivity & Superadditivity

☞ **Subadditivity**

\[ H(t_1) + H(t_2) \geq H(t_1 + t_2) \quad \forall \; t_1, t_2 \geq 0 \quad \text{and} \quad 0 \leq t_1 + t_2 \leq T \]

(The number of events in an interval of the form \([0, t_2]\) is larger than the number of events taking place in an interval of the same length beginning later (i.e. in the form of \([T, T+t_2]\))

☞ The number of failures is decreasing

☞ **Superadditivity**

\[ H(t_1) + H(t_2) \leq H(t_1 + t_2) \quad \forall \; t_1, t_2 \geq 0 \quad \text{et} \quad 0 \leq t_1 + t_2 \leq T \]

(The number of events in an interval of the form \([0, t_2]\) is smaller than the number of events taking place in an interval of the same length beginning later (i.e. in the form of \([T, T+t_2]\))

☞ The number of failures is increasing

---

Trend tests

☞ **Means**

- Raw data ⇒ graphical tests
- Analytical tests ⇒ quantitative indicators

☞ **Raw data**

- Times to successive failures
- Number of failures per unit of time
- Cumulative number of failures

☞ **Trend indicators**

- Empirical (arithmetical) means
- Subadditivity factor
- Laplace factor
Graphical tests: times to failures (Example A)

Times to failures
\( t_1 \)

Cumulative times to failures
\( t_1 + \ldots t_k \)

Graphical test: grouped data (Example B)

Failure intensity

Cumulative number of failures

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Empirical mean

Global trend

ξₖ : arithmetical mean of the times to failures (from failure 1 to k)

\[ \xi_k = \frac{\sum_{i=1}^{k} t_i}{k} \]

ξₖ constitute a globally increasing series ⇔ reliability growth

ξₖ constitute a globally decreasing series ⇔ reliability decrease

The trend is directly observed on the evolution of ξₖ

Example A

local trend

• The data items are grouped into subsets containing m successive data
• The average is evaluated for each subset
• The impact of old data items is eliminated

Example A: m = 8 ⇒ 17 groups (136 failures)
Subadditivity factor

Graphical interpretation of subadditivity
- $H(t) = E[N(t)]$ is subadditive over $[0, T]$ if:
  
  $$a_H(t) = \int_0^t H(x) \, dx - \frac{1}{2} H(t) \geq 0$$
  
  $\forall \, t \geq 0 \text{ and } 0 \leq t \leq T$

$a_H(t)$ = subadditivity factor

Subadditivity & monotonous growth / decrease

Monotonous growth:
- $a_H(x) > 0$ increasing

Monotonous decrease:
- $a_H(x) < 0$ decreasing

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Subadditivity & local trend fluctuations

Example: reliability growth with local fluctuations

\[ a_N(x) \geq 0 \text{ non decreasing} \]

Subadditivity & trend change

Decrease - Growth

Growth - Decrease

\[ a_N(x) < 0 \quad \text{or} \quad a_N(x) > 0 \]
Laplace factor

☞ Statistical Test of hypothesis ⇒ Laplace factor $u$

Random variable: times to failures $T_i$ (realization of $T_i = t_i$)

$$u(T) = \frac{1}{N(T)} \sum_{i=1}^{N(T)} \sum_{j=1}^{i} t_j - \frac{T}{2}$$

$N(T) = \#$ failures in $[0,T]$

Random variable: $\#$ failures per unit of time

$$u(T) = \frac{\sum_{i=1}^{k} (i-1) n(i) - \frac{k-1}{2} \sum_{i=1}^{k} n(i)}{\sqrt{\frac{k(k-1)}{12} \sum_{i=1}^{k} n(i)}}$$

$n(i) = \#$ failure during time unit $i$

☞ In practice

$u > 0 \Rightarrow$ global reliability decrease

$u < 0 \Rightarrow$ global reliability growth

Interpretation of the Laplace test

☞ Random Variable: time to failure

• $T/2 = \text{mid of the observation interval}$

• $c = \frac{1}{N(T)} \sum_{i=1}^{N(T)} \sum_{j=1}^{i} t_j : \text{statistical centre}$

$$\frac{T}{2} > c \Rightarrow \text{reliability growth}$$

☞ Random Variable: $\#$ failures per unit of time

• The Laplace factor can be put in the form:

$$u(T) = -a_{H}(T)$$

$$T \sqrt{\frac{T}{12 N(T)}}$$

⇒ Relationship between Laplace and subadditivity factors
Link: graphical tests - Laplace - Subadditivity

Failure intensity

Cumulative number of failures

Subadditivity factor

Laplace factor

Laplace factor: local and global trend

GLOBAL TREND

GLOBAL

Reliability decrease

Reliability growth

Local trend changes

LOCAL TREND

Reliability decrease

Reliability growth

Reliability decrease
Change of time origin

Monotonous Growth

Monotonous decrease

Decrease followed by growth

Stability

Link between trend indicators

Cumulative number of failures | Failure intensity | Laplace factor
--- | --- | ---
Monotonous Growth | ![Graph](image1) | ![Graph](image2) | ![Graph](image3)
Monotonous decrease | ![Graph](image4) | ![Graph](image5) | ![Graph](image6)
Decrease followed by growth | ![Graph](image7) | ![Graph](image8) | ![Graph](image9)
Stability | ![Graph](image10) | ![Graph](image11) | ![Graph](image12)
How to use trend test results

☞ Control of the efficiency of test activities
  • Reliability decrease at the beginning of a new activity: OK
  • Reliability decrease during a relatively long period of time: Pb?
  • Reliability growth after reliability decrease: OK
  • Sudden reliability growth: caution!
  • Stable reliability: saturation
    ☞ New tests
    ☞ Following phase
    ☞ End of test

☞ Application of reliability models
  • Trend in accordance with model assumptions

Application to RADC data sets

*Rome Air Development Center (USA)*

<table>
<thead>
<tr>
<th>System id.</th>
<th># instructions</th>
<th># programmers</th>
<th># failures</th>
<th>Type of system</th>
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<tr>
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<tr>
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</tbody>
</table>

RT: Real-time
C: control
Com.: commercial
WP: word Processing
TS: Time sharing
OS: Operating system
*****: not given

[John Musa], “Software Reliability Data”, Rome Air Development Center, NY, USA, 1979
Laplace factors

<table>
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<td>- 5,64</td>
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*: stable reliability

System 2: times to failures

Time to failures, \( t_i \)

# failures

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System 2: Laplace factor

Variable: time to failure

System 2: failure intensity

k: unit of tims = 5000 seconds of execution time
System 2: Laplace factor

Variable: # failures

Unit of time = 5000 seconds of execution time

System 4: arithmetical mean

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Conclusion

- Some systems can be modeled by an exponential distribution
  - System for which $-2 < u < 2$

- Influence of the operational profile
  - Systems 11 A, B, C are 3 copies of the same program used in different environments

- Benefits from trend analysis
  - Better understanding of the underlying processes
  - Follow up of the development process in real-time, fast feedback
  - Helpful for reliability model application