

# Computational Metaphysics

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## Metaphysics

- The word comes from Andronicus of Rhodes, who edited Aristotle's works 100 years after his death
- He grouped some of the works as “the physical ones” and 14 left over were called “the after the physical ones”
  - Ta meta ta phusika, or metaphysics
- Aristotle himself used terms like being as such, first causes, that which does not change for the topics concerned
- With the rise of science, many topics in physics were reassigned to new disciplines
  - Chemistry, modern physics, cosmology, genetics and the leftovers went to metaphysics
  - e.g., free will, mind-body duality
- The new disciplines are based on mathematical models
  - Predictive, testable, practical applications
- Maybe it's time to do the same for metaphysics

# Computational Metaphysics

- Proposal by Fitelson and Zalta (Stanford)
  - Build mathematical model of some metaphysical topic
  - Calculate consequences
- The models are primarily collections of axioms
  - So the relevant mathematical discipline is **logic**
  - And calculation is **automated deduction**
- More specifically
  - Code a metaphysical topic in a mechanized logic
  - Let the automation rip
  - Examine the result for insights
- An example: **The Ontological Argument**
  - This is a proof of the existence of God!

# Classical Arguments for Existence of God

**Teleological:** argument from design

This is an **empirical** or *a posteriori* argument: it builds on empirical observations about the world

Hence is vulnerable to better understanding of empiricism, better observations, better explanations

- Hume, Darwin etc.

**Cosmological:** there must be a first (uncaused) cause

Or **why is there something rather than nothing?**

Also *a posteriori*, but less reliant on specifics

But depends on notion of cause

- Leibniz, Hume, Kant; current popularization: Holt

**Ontological:** next slide

This is a **rational** or *a priori* argument: it doesn't depend on observation

## The Ontological Argument (St. Anselm, 11th C)

**Thus** even the fool is convinced that something than which nothing greater can be conceived is in the understanding, since when he hears this, he understands it; and whatever is understood is in the understanding.

**And** certainly that than which a greater cannot be conceived cannot be in the understanding alone.

**For** if it is even in the understanding alone, it can be conceived to exist in reality also, which is greater.

**Thus** if that than which a greater cannot be conceived is in the understanding alone, then that than which a greater cannot be conceived is itself that than which a greater can be conceived.

**But** surely this cannot be.

**Thus** without doubt something than which a greater cannot be conceived exists, both in the understanding and in reality.

## The Ontological Argument: Modern Reading

- We can conceive of something than which there is no greater
  - A variant is “a being that possesses all perfections”
- If that thing does not exist in reality, then we can conceive of a greater thing—namely, something that does exist in reality
- Therefore either the greatest thing exists in reality or it is not the greatest thing
- Therefore the greatest thing necessarily exists in reality
- That’s God
  - Why it’s the Christian God is another matter
  - Seems more like the Neo-Platonist “One”
  - Or Spinoza’s “God or Nature”

## Status of The Ontological Argument

- Formulated by **St. Anselm** (1033–1109)
  - Archbishop of Canterbury
  - Aimed to justify Christian doctrine through reason
- Disputed by his contemporary Gaunilo
  - Existence of a **perfect island**
- Widely studied and disputed thereafter
- Descartes (used in the Cogito, several variants), Leibniz, Hume, Kant (who named it), Gödel
- Russell, on his way to the tobacconist: “**Great God in Boots!—the ontological argument is sound!**”
- Ridiculed, but in trivialized form, by Dawkins and others
- The later Russell: “**The argument does not, to a modern mind, seem very convincing, but it is easier to feel convinced that it must be fallacious than it is to find out precisely where the fallacy lies**”

## Significance of The Ontological Argument

- Almost everyone finds this topic interesting
- Believers and unbelievers alike
  - Many of those who studied and criticized the Argument were devout believers
  - Can something as ineffable as the existence of God can be subject to a mere *a priori* demonstration?
- The proof raises quite deep issues in logic
  - How to handle definite descriptions that may not denote
    - ★ e.g., The present King of France is bald
  - Is the proof logically sound?
- And in the interpretation of logical proofs
  - What are the assumptions, and do we believe them?
  - What does the proof really establish?
- Just like formal methods in support of Safety Cases



## Logic of the Ontological Argument

- Anselm himself gave two variants of the Argument
- The second asserts not the mere possibility that a maximally great something exists, but that it **necessarily** exists
- So several modern treatments use **modal logics**
  - Gödel, Plantinga
- Oppenheimer and Zalta make a good case that the basic argument can/should be interpreted in **classical logic**, but we need to be careful about **existence**

# Existence

Two issues:

**Existence in reality:** this is not the same as  $\exists$ , which although it is pronounced “there exists” refers to an implicit domain of quantification and does not assert existence in reality (think “not  $\forall$  not” )

**Quantifiers ranging over possibly nonexistent objects:** can lead to unsoundness in first order logic

Oppenheimer and Zalta use **Free Logic**, which has an explicit existence predicate ( $E!$ ) and adjusts the quantifier rules

## Logic of the Ontological Argument (ctd.)

- The argument uses a definite description
  - The  $x$  such that some property  $\phi$ :  $\iota x\phi$
  - Here, “that (i.e., the  $x$ ) than which there is no greater”
- These are tricky
  - “The present King of France is bald”
    - ★ Note, for those who learn about the world from CNN or the WSJ: France is a republic, it has no present king
  - Is this true, false, inadmissible?
  - If the former, its negation should be false
  - What **is** its negation?
- Related to the existence problem
  - Must not substitute definite descriptions into quantified expressions without being sure they are well defined

## Oppenheimer and Zalta's Treatment

- Careful treatment in **unmechanized Free Logic**, 1991
- The treatment was later mechanized in **Prover9**, 2011
- Claimed that Prover9 discovered a simplification to the original that  
“not only brings out the beauty of the logic inherent in the argument, but also clearly shows how it constitutes an early example of a ‘diagonal argument’ used to establish a positive conclusion rather than a paradox”
- Prover9 uses **classical** First Order Logic
  - Not a Free Logic, lacks definite descriptions
  - So there's manual reformulation
- Garbacz claims the simplification is due to **unsoundness** in this reformulation

## The Ontological Argument in PVS

- I recently formally verified the Argument using PVS
  - A higher order logic
    - ★ With dependent typing and predicate subtypes
  - Provides sound and mechanically enforced treatment of existence and quantification, definite descriptions, and much else
- PVS shows the argument is sound!

## Assumptions, Conclusions

- PVS verification depends on four assumptions
  1. Trichotomy of the “ordering”
  2. Existence of at least one “greatest thing”
  3. Something exists in reality
  4. Things that exist in reality greater than those that do not
- 1 and 2 ensure “that than which no greater” is **defined**
- 3 and 4 ensure it exists in **reality**
- Usual formulation of 3 and 4 render it **circular**
- Demonstrate consistency of axiomitization by exhibiting constructive model using **theory interpretations** in PVS
  - Interpret **beings** by the natural numbers **nat**
  - And **>** by **<** (so **the(greatest)** is **0**)
  - And **really\_exists** by “**less than 4**”
- This is not the **intended** interpretation!

## Summary

- Have **formalized** the Ontological Argument
- And **verified** its conclusion
- So the Argument is **sound!**
- But it is **very close to circular**
  - And slight variants **are** circular
- And it **does not compel the intended interpretation**
- I think it is a **Fun** example to introduce students to
  - Subtle issues in **logic** and **mechanization**
  - The **interpretation** and utility of formally verified claims

## Homework

- Subject the dozens of **variants** of the Argument to the same examination: are the differences significant?
- Reconstruct **Gödel's** or Plantinga's proofs in PVS
  - Will need to embed a modal logic (which one?) in PVS
  - Embedding of LTL (S4) could serve as a model
  - . . . actually not, needs full first-order modal logic
- **Hot news!** Benzüller and Woltzenlogel-Paleo have done this (in Isabelle and Coq)
- Try to formalize and verify **Avicenna's** proof of the “**Necessary Existent**”
  - Older than the Ontological Argument
  - And arguably less of a logical “trick” and closer (for some) to the true source of belief
- Figure out how to **monetize** this



## A Crazy Idea: Computational Philosophy

- Fitelson and Zalta proposed “computational metaphysics”
- I think this is reasonable, but **too modest**
- A lot of philosophy is implicitly based on an anthropomorphic interpretation of knowledge, learning, deduction, language, communication, etc.
- As computer scientists we have a **unique** grasp of computational interpretations of these
  - From AI, robotics, machine learning, etc.
  - Cf. Searle’s Chinese Room: he just doesn’t get it
- I think this creates a potential for new insights on traditional philosophical questions
  - Consciousness, rationality, free will, ethics, emergence
- Why this matters: for advanced AI (a bit beyond the Google Car) these become issues of **practical concern**

## Some Suggested Reading

- Oppenheimer and Zalta's papers: just Google for them
- [36 Arguments for the Existence of God: A Work of Fiction](#) by Rebecca Goldstein
- [Types, Tableaus, and Gödel's God \(Trends in Logic\)](#) by Mel Fitting
- [Why Does The World Exist? An Existential Detective Story](#) By Jim Holt
  - See also Freeman Dyson's review in NY Review of Books