What Does Verification Achieve?

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Software Correctness vs. System Claims

- The top-level requirements for most complex systems are stated quantitatively
    - E.g., no catastrophic failure in the lifetime of all airplanes of one type
- And these lead to probabilistic requirements for software-intensive subsystems
    - E.g., probability of failure in flight control less than 10^{-9} per hour
- But verification in general
    - And formal verification in particular

  Are about correctness. . . an absolute notion
- How do we connect the absolute claims of verification for software with probabilistic requirements at the system level?

Software Reliability

- Software contributes to system failures through faults in its requirements, design, implementation—bugs
- A bug that leads to failure is certain to do so whenever it is encountered in similar circumstances
    - There's nothing probabilistic about it
- Aaah, but the circumstances of the system are a stochastic process
- So there is a probability of encountering the circumstances that activate the bug
- Hence, probabilistic statements about software reliability or failure are perfectly reasonable
- Typically speak of probability of failure on demand (pfd), or failure rate (per hour, say)

Measuring/Predicting Software Reliability

- For pfds down to about 10^{-4}, it is feasible to measure software reliability by statistically valid random testing
- But 10^{-9} would need 114,000 years on test
- So how do we establish that a piece of software is adequately reliable for a system that requires, say, 10^{-6}?
- Most standards for system safety (e.g., IEC 61508, DO178B) require you to show that you did a lot of V&V
  - e.g., 57 V&V "objectives" at DO178B Level C (10^{-5})
- And you have to do more for higher levels
  - 65 objectives at DO178B Level B (10^{-7})
  - 66 objectives at DO178B Level A (10^{-9})

Does "More Correct" Mean More Reliable?

- These V&V objectives are all about correctness
  - Requirements tracing, testing etc.
- More V&V objectives might make the software "more correct" but what does that have to do with reliability?
- And what does "more correct" mean anyway?

Possibly Perfect Software

- Instead of correct software
  - Which is about conformance with specification
- We'll speak of perfect software
  - Software that will never experience a failure in operation, no matter how much operational exposure it has
- You might not believe a given piece of software is perfect
- But you might concede it has a possibility of being perfect
- And the more V&V it has had, the greater that possibility
- So let's speak of a probability of perfection
  - Think of all the software that might have been developed by comparable engineering processes to solve the same design problem as the software at hand
  - The probability of perfection is then the probability that any software randomly selected from this class is perfect

Probabilities of Perfection and Failure

- Probability of perfection relates to correctness-based V&V
- And it also relates to reliability:

  By the formula for total probability

$$P(\text{s/w fails [on a randomly selected demand]}) \quad (1)$$
$$= P(\text{s/w fails} \mid \text{s/w perfect}) \times P(\text{s/w perfect})$$
$$+ P(\text{s/w fails} \mid \text{s/w imperfect}) \times P(\text{s/w imperfect}).$$

  - The first term in this sum is zero, because the software does not fail if it is perfect
  - Can then, very conservatively, assume that the software always fails if it imperfect, so that the first factor in the second term becomes 1 (more exact treatment later)

  Hence, very crudely, and very conservatively,

$$P(\text{software fails}) < P(\text{software imperfect}) \quad (2)$$

Two Channel Systems

- Many safety-critical systems have two (or more) diverse "channels"
  - E.g., nuclear shutdown, flight control
- One operational channel does the business
- A simpler channel provides a backup or monitor
- Cannot simply multiply the pfds of the two channels to get pfd for the system
  - Failures are unlikely to be independent
  - Failure of one channel suggests this is a difficult case, so failure of the other is more likely
  - Infeasible to measure amount of dependence

Two Channel Systems and Possible Perfection

- But if the second channel is possibly perfect
  - Its imperfection is conditionally independent of failures in the first channel
  - Hence, system pfd is conservatively bounded by product of pfd of first channel and probability of imperfection of the second
- Joint work with Bev Littlewood:
  `http://www.csl.sri.com/~rushby/abstracts/csl-09-02`
  - Who originated the idea of possible perfection
- May provide justification for some of the architectures suggested in ARP 4754
  - e.g., 10^{-9} system made of Level C operational channel and Level A monitor

Aleatory and Epistemic Uncertainty

- Aleatory or irreducible uncertainty
  - is "uncertainty in the world"
  - e.g., if I have a biased coin with $P(heads) = p_h$, I cannot predict exactly how many heads will occur in 100 trials because of randomness in the world

  Frequentist interpretation of probability needed here

- Epistemic or reducible uncertainty
  - is "uncertainty about the world"
  - e.g., if I give you the biased coin, you will not know p_h; you can estimate it, and can try to improve your estimate by doing experiments, learning something about its manufacture, the historical record of similar coins etc.

  Frequentist and subjective interpretations OK here

Aleatory and Epistemic Uncertainty in Models

- In much scientific modeling, the aleatory uncertainty is captured conditionally in a model with parameters
- And the epistemic uncertainty centers upon the values of these parameters
- As in the coin tossing example
- Analysis in (1) was aleatory, with parameters
    - p_{np} probability the software is imperfect
    - p_{fnp} probability that it fails, if it is imperfect
    - $P(\text{software fails}) < p_{fnp} \times p_{np}$

Epistemic Estimation

- To apply this result, we need to assess values for p_{fnp} and p_{np}
- These are most likely subjective probabilities
  - i.e., degrees of belief
- Beliefs may not be independent
- So will be represented by some joint distribution $F(p_{fnp}, p_{np})$
- Probability of system failure will be given by the Riemann-Stieltjes integral

$$\int_{\substack{0 \le p_{fnp} \le 1 \\ 0 \le p_{np} \le 1}} p_{fnp} \times p_{np} \, dF(p_{fnp}, p_{np}). \quad (3)$$

- If beliefs can be separated F factorizes as $F(p_{fnp}) \times F(p_{np})$
- And (3) becomes $P_{fnp} \times P_{np}$

  Where these are the means of the posterior distributions representing the assessor's beliefs about the two parameters

Formal Verification and the Probability of Perfection

- We want to assess P_{np}
- Context is likely a safety case in which claims about a system are justified by an argument based on evidence about the system and its development
- Suppose part of the evidence is formal verification
- ◦ i.e., what is the probability of perfection of formally verified software?
- This is considered in the paper with Bev, and in
  `http://www.csl.sri.com/~rushby/abstracts/sefm09`

Application

- Suppose we need P_{np} of 10^{-4}
- Bulk of this "budget" should be divided between incorrect formalization and incompleteness of the formal analysis, with small fraction allocated to unsoundness of verification system
- Through sufficiently careful and comprehensive formal challenges, it is plausible an assessor can assign a subjective posterior probability of imperfection on the order of 10^{-4} to the formal statements on which a formal verification depends
- Through testing and other scrutiny, a similar figure can be assigned to the probability of imperfection due to discontinuities and incompleteness in the formal analysis
- By use of a verification system with a trusted or verified kernel, or trusted, verified, or diverse checkers, assessor can assign probability of 10^{-5} or smaller that the theorem prover incorrectly verified the theorems that attest to perfection

Discussion

- These numbers are feasible and plausible
- Formal methods and their tools do not need to be held to (much) higher standards than the systems they assure
- But what are we to do about single channel systems that require 10^{-9}?
  - Topic for investigation and discussion whether such assessments could be considered feasible and credible
  - The accuracy of the properties checked dominates accuracy of the checking

Conclusion

- Probability of perfection is a radical and valuable idea
- Provides the bridge between correctness-based verification activities and probabilistic claims needed at the system level
- Relieves formal verification, and its tools, of the burden of absolute perfection